

Parametric Excitation of Shear Waves in Soft Solids

M. A. Mironov^a, P. A. Pyatakov^a, I. I. Konopatskaya^a, G. T. Clement^b, and N. I. Vykhotseva^b

^a*Andreev Acoustics Institute, Russian Academy of Sciences,
ul. Shvernika 4, Moscow, 117036 Russia
e-mail: mironov@akin.ru*

^b*Department of Radiology, Brigham and Women's Hospital, and Harvard Medical School,
75 Francis Street, Boston, MA 02115, USA
e-mail: Natalia@bwh.harvard.edu*

Received February 9, 2009

Abstract—One of the possible mechanisms that may underlie ultrasound-induced damage of soft solids is discussed. The model of three-wave interaction in a solid is used to consider shear wave generation by a longitudinal sound wave in a solid with a small shear modulus. Numerical estimates are obtained for the excitation threshold of the shear wave in biological tissues. Since the wavelength of ultrasound-generated shear waves is small, the shear stresses may be sufficient to destroy the structure of biological tissue. Results of model experiments are presented.

PACS numbers: 43.25.Dc, 43.25.Lj, 43.35.Cg, 43.80.Gx

DOI: 10.1134/S1063771009040137

The interest in shear waves propagating in soft solids has quickened after the appearance of two papers devoted to this subject [1, 2]. In these papers, the authors proposed to carry out elastographic studies of biological tissues with the use of shear waves generated by modulated radiation pressure of focused ultrasound. When the force of the radiation pressure caused by absorption or scattering in the focal region of the radiating transducer is modulated with a low frequency, it gives rise to shear waves outgoing from the focal region. The propagation velocity of shear waves can be used to determine the Young modulus of the tissue, which characterizes the state of the latter. Today, one can find many publications devoted to such “sonoelastographic” studies of biological tissues (see, e.g., [3]). For example, one of the most recent papers demonstrates the possibility to detect the inhomogeneities of the shear modulus due to heating [4].

Nonlinear wave interactions in such media have also been widely investigated. Unlike liquids, where only longitudinal waves can interact, in elastic media, shear waves can also take part in wave interactions.

For the five-constant elasticity theory describing an isotropic medium [5] with only quadratic nonlinearity taken into account, equations determining all the possible wave interactions were derived in [6]. Classification of interacting waves and the conditions on frequencies and wave numbers that allow the interaction of plane waves (resonance conditions) were given in [7]. In the same paper, the authors calculated the scattered fields generated by beams of interacting waves with the use of the small perturbation method. Nonlinear processes in shear fields where only odd nonlin-

earity is possible were theoretically considered in [8] where equations for a beam of a shear wave were derived with allowance for nonlinearity and parabolic transverse diffusion, and in [9], where a self-similar transformation was proposed and the self-focusing was considered in the approximation of geometric acoustics. In [10], a modified representation of the potential energy of strain was proposed to separate the contributions due to uniform compression and shear. Beams of shear waves with linear and elliptic polarizations were theoretically studied in [11]. An experimental observation of a two-frequency interaction between shear waves was reported in [12]. In other experiments [13], both a two-frequency interaction and odd harmonic generation were observed for plane waves. In [14], the formation of shear shock waves was detected when the Mach number with respect to the shear wave's velocity was about 0.4. Finally, in [15], an attempt was made to determine the nonlinear moduli from the measured dependence of shear wave's propagation velocity on static strain in a soft solid specimen. The authors of [15] also derived relations between the shear wave velocity and the constant A of the five-constant elasticity theory.

In the present paper, we consider the least studied type of wave interaction in soft solids: the decay of a longitudinal sound wave into two shear waves. This is a particular case of interaction between a longitudinal wave and a shear wave, which leads to generation of a transverse wave. In the table of allowed interactions that was given in [7], this type of interaction corresponds to the fifth row. Figure 1 shows the diagram of the corresponding interaction. The initial sound wave

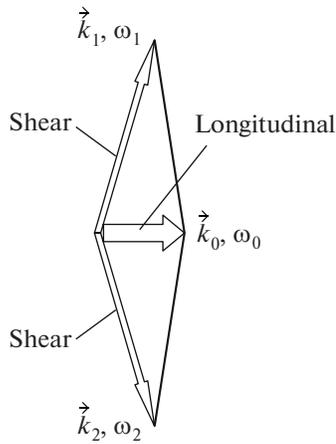


Fig. 1. Diagram illustrating the decay of a longitudinal (sound) wave into two shear waves.

with the wave vector k_0 and frequency ω_0 falls into two shear waves with the wave numbers $\vec{k}_{1,2}$ and frequencies $\omega_{1,2}$. The wave vectors and the frequencies satisfy the relations

$$\vec{k}_0 = \vec{k}_1 + \vec{k}_2; \quad \omega_0 = \omega_1 + \omega_2. \quad (1)$$

The characteristic feature of soft solids consists in that the velocity of the transverse wave in them is two or three orders of magnitude smaller than the velocity of the longitudinal (sound) wave. Therefore, the magnitudes of the wave vectors of the generated shear waves are much greater than the magnitude of the wave vector of the initial sound wave; hence, the shear waves propagate in the directions perpendicular to the direction of sound wave propagation. The purpose of our paper is to attract the attention of specialists in medical and biological acoustics to this mechanism of transfer of sound energy to small-scale vibrations of the medium as an alternative of the thermal and cavitation mechanisms of ultrasonic action upon biological media.

It is commonly believed that ultrasound-induced damage of biological tissues is caused by either heating or cavitation. In addition, it is possible that ultrasound destroys the tissue because of purely mechanical effects [16–19]. In [16–18], the absence of cavitation was achieved by applying sufficiently high static pressure. A review of various effects that may be significant for biological action of ultrasound can be found in [20] (Chapters 12 and 13).

Histological observations carried out by one of us (N.I. Vykhodtseva) [21], as well as some other investigations, suggest that alternative mechanisms of damage caused by ultrasound should be considered. As an example of ultrasonic action, Fig. 2 shows microscopic sections of a rabbit's brain tissue affected by focused ultrasound with different parameters. The left-hand plot represents the homogenized tissue after the action of focused ultrasound (the total radiation power is 360 W, the frequency is 1.63 MHz, the pulse duration is 0.005 s, the pulse's repetition frequency is 2 Hz, and the total duration of ultrasonic action is 10 s); the right-hand plot shows the cavitation-induced damage obtained by applying a twofold ultrasonic power. A detailed description can be found in [22].

So far, no adequate explanations of nonthermal and noncavitation damage of tissue in an ultrasonic field have been found. The shear waves generated by sound as a result of decay interaction can be considered as a possible origin of the purely mechanical, noncavitation damage of tissue.

For the three-wave interaction involving one longitudinal and two shear waves, calculations can be performed with the use of the formulas derived in [7]. However, below, we use a simplified approach, which takes into account that, in a soft solid, the wavelength of sound considerably exceeds the wavelength of shear waves. This allows us to ignore the spatial structure of the sound wave and to assume that shear waves propagate in the medium, in which the pressure (or the uniaxial strain or stress) varies periodically in time and synchronously at all of the spatial points. One more

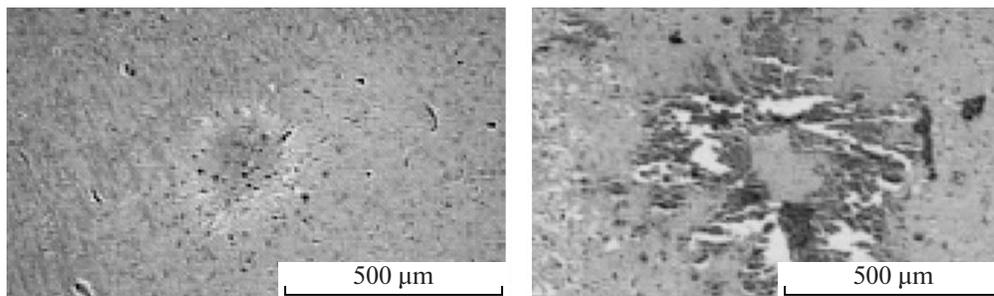


Fig. 2. Histological sections of rabbit's brain tissue subjected (in vivo) to the action of focused ultrasound with different parameters. The left-hand plot represents homogenized tissue after the action of focused ultrasound (with a total radiation power of 360 W, a frequency of 1.63 MHz, a pulse duration of 0.005 s, a pulse repetition frequency of 2 Hz, and a total action duration of 10 s). The right-hand plot displays cavitation damage arising under the effect of approximately twofold ultrasonic power.

simplification consists in neglecting variations in the density of the medium.

We write the wave equation for the displacement $\bar{u}(r, t)$ in shear waves with allowance for the pressure dependence of the shear modulus:

$$\rho_0 \frac{\partial^2 \bar{u}}{\partial t^2} - \mu(t) \Delta \bar{u} = 0. \quad (2)$$

The shear modulus μ depends on the pressure $p(t)$ as

$$\mu(t) = \mu_0 [1 + \alpha \cdot p(t)], \quad (3)$$

where α is a positive constant determining this dependence. Note that, the pressure dependence could be replaced by the dependence on uniaxial stress if the results obtained in [15] were used.

The inverse effect of transverse waves on sound is ignored; in other words, we assume that the pressure $p(t)$ is a given function. Without going beyond the quadratic approximation, we represent Eq. (2) in the form

$$\rho_0 \frac{\partial^2 \bar{u}}{\partial t^2} - \mu_0 \Delta \bar{u} = \rho_0 \cdot \alpha \cdot p(t) \cdot \frac{\partial^2 \bar{u}}{\partial t^2}. \quad (4)$$

We preset the sound pressure in the form of a harmonic plane traveling wave with an amplitude P_0 , a frequency ω_0 , and a wave vector \vec{k}_0 :

$$p = P_0 \sin(\omega_0 t - \vec{k}_0 \cdot \vec{x}). \quad (5)$$

In compliance with the theory of a resonant three-wave interaction, we seek the field of the shear wave in the form of a pair of traveling waves with identical time-dependent amplitudes:

$$\bar{u}(t, \vec{x}) = \bar{A}(t) \cdot [\sin(\omega t - \vec{k}_1 \cdot \vec{x}) + \sin(\omega t - \vec{k}_2 \cdot \vec{x})]. \quad (6)$$

The frequencies and the wave numbers satisfy the conditions of synchronism given by Eqs. (1).

Now, we use the small perturbation method: we multiply Eq. (5) by the second time derivative of Eq. (6) and substitute the result on the right-hand side of Eq. (4), which we consider as an additional source generating the additional shear field. Multiplying the trigonometric functions, we obtain the difference between the cosines with sum and difference frequencies $\omega_0 \pm \omega_{1,2}$ and with wave numbers $\vec{k}_{1,2}$. Using condition (1), on the right-hand side we retain only the difference terms, which are synchronous with the left-hand side of Eq. (4):

$$\rho_0 \cdot \alpha \cdot p \cdot \frac{\partial^2 \bar{u}}{\partial t^2} = \rho_0 \cdot \alpha \cdot P_0 \cdot \bar{A} \cdot \{\omega_1^2 \cos(\omega_2 t - \vec{k}_2 \cdot \vec{x}) + \omega_2^2 \cos(\omega_1 t - \vec{k}_1 \cdot \vec{x})\}. \quad (7)$$

We additionally simplify our calculations by assuming that the magnitude of the wave number of the sound wave is much smaller (by a factor of 100 or 1000) than

the magnitudes of the wave numbers of shear waves.

Then, $\omega_1 = \omega_2 = \frac{1}{2} \omega_0$ and Eq. (7) takes the form

$$\rho_0 \cdot \alpha \cdot p \cdot \frac{\partial^2 \bar{u}}{\partial t^2} = \rho_0 \cdot \alpha \cdot P_0 \cdot \bar{A} \omega_1^2 \{\cos(\omega_2 t - \vec{k}_2 \cdot \vec{x}) + \cos(\omega_1 t - \vec{k}_1 \cdot \vec{x})\}.$$

Comparing the space and time dependences of the right-hand side of this expression with expression (6) for the shear field, we see that the right-hand side of Eq. (4) can be represented as

$$\rho_0 \cdot \alpha \cdot p \cdot \frac{\partial^2 \bar{u}}{\partial t^2} = \rho_0 \cdot \alpha \cdot P_0 \cdot \omega_1 \frac{\partial \bar{u}}{\partial t}. \quad (8)$$

Rearranging it to the left-hand side of Eq. (4), we obtain

$$\rho_0 \frac{\partial^2 \bar{u}}{\partial t^2} - \rho_0 \cdot \alpha \cdot P_0 \cdot \omega_1 \cdot \frac{\partial \bar{u}}{\partial t} - \mu_0 \Delta \bar{u} = 0. \quad (9)$$

Thus, the effective force caused by nonlinear interaction of sound with the shear field is proportional to the displacement velocity in the shear field, $\frac{\partial \bar{u}}{\partial t}$. This is the equivalent of the negative force of external friction with friction coefficient γ , which is determined as $\gamma = -\rho_0 \cdot \alpha \cdot P_0 \cdot \omega_1$. The presence of negative friction leads to an exponential increase in the amplitude of shear waves: $\bar{A}(t) = \bar{A}_0 \exp(\delta \cdot t)$, where the increment δ is

$$\delta = \frac{1}{2} \cdot \alpha \cdot P_0 \cdot \omega_1 = \frac{1}{4} \cdot \alpha \cdot P_0 \cdot \omega_0. \quad (10)$$

Thus, the dependence of the shear modulus of the soft solid on pressure leads to a parametric build-up of shear waves in the field of the sound wave. The frequency of the shear waves ω_1 makes half the frequency of the sound wave ω_0 . If we assume that the propagation velocity of shear waves in soft biological tissues is approximately 1000 times smaller than the propagation velocity of longitudinal waves, the wavelength of the shear wave excited by a sound wave whose frequency is 1 MHz should be 3 μm .

To take into account the regular (linear) absorption of the shear wave, we subtract the linear damping coefficient ε from the right-hand side of Eq. (10):

$$\delta = \left(\frac{1}{2} \cdot \alpha \cdot P_0 - \varepsilon \right) \cdot \omega_1. \quad (11)$$

The presence of absorption gives rise to an instability threshold: the exponential build-up is only possible when the amplitude of the sound pressure exceeds the threshold value

$$P_0^* = 2 \frac{\varepsilon}{\alpha}. \quad (12)$$

If the pressure amplitude far exceeds the threshold, the characteristic time of the shear wave's amplitude growth (the build-up time) τ is estimated as

$$\tau = 2/(\alpha \cdot P_0 \cdot \omega_0) = 4/(\alpha \cdot P_0 \cdot \omega_0). \quad (13)$$

Above, we considered only one pair of shear waves interacting with the longitudinal wave. By virtue of axial symmetry, pairs of wave numbers of shear waves can be rotated through any angle about the wave vector of the sound wave. Hence, when the instability threshold is exceeded, the insonified region becomes filled with shear waves traveling in all the directions that are approximately orthogonal to the direction of the sound wave's propagation.

To estimate the magnitude of the effects, it is necessary to know the values of the coefficients α and ε . An experimental study of the effect of static uniaxial stress on the propagation velocity of shear waves with a frequency of 50 Hz in agar-gelatin gel was described in [15]. The dependences obtained in these experiments allow the determination of the nonlinearity coefficient α . This coefficient was found to be not only specimen-dependent, but also dependent on the polarization of shear waves, and its values varied within $\alpha \sim 10^{-4} - 2 \times 10^{-3} \text{ Pa}^{-1}$. In principle, this value can be used to estimate the parametric generation of shear waves. However, intuition suggests that the values of α obtained in [15] are overestimated. We take the value of α to be several orders of magnitude smaller, for example, $\alpha = 10^{-7} \text{ Pa}^{-1}$ (the shear modulus increases by a factor of two when the pressure increases to 100 atm). For the parameter ε , we take the value $\varepsilon = 10^{-1}$. Then, the amplitude of the threshold pressure given by Eq. (12) will be $P_0^* = 2 \times 10^6 \text{ Pa}$, which approximately corresponds to a flux density of acoustic energy of 130 W/cm^2 . The characteristic build-up time in the case of a twofold excess over the threshold is (see Eq. (13)) $\tau = 5/\omega_0$. When the frequency of sound is 1 MHz, this time is $\tau \sim 10^{-6} \text{ s}$.

Evidently, to obtain more accurate estimates, it is necessary to determine the coefficients α, ε experimentally; moreover, the values of these parameters should be determined in the megahertz frequency range.

The theory presented above describes only the initial stage of the process: the instability threshold and the beginning of exponential increase. The increase cannot last without a bound. To adequately describe the subsequent evolution, it is necessary to consider the constraining effects. The first step is to take into account the inverse influence of shear waves on the sound wave. In classical nonlinear optics, this influence is described by the Bloembergen equations [23]. In their pure form, these equations describe the cyclic energy transfer between three interacting waves. In our case, they should be complemented with the loss in

shear waves, the higher harmonic generation, and the generation of a shock wave [9, 11, 13, 14]. As a result, we may obtain a steady-state process in which the shear wave amplitude is maintained at a constant level due to the balance between the energy transfer from the sound wave and the energy loss in shear waves. For the sound wave, this manifests itself in an effective nonlinear loss. A more significant and interesting effect is the influence of the structure damage on the initial parametric interaction. The main physical result of the three-wave interaction under study may consist in the partial damage of the structure of the solid or in a total failure of this structure. This leads to an increase in the loss coefficient characterizing the energy loss in shear waves. An increase in the damping coefficient leads to suppression of instability: the energy transfer from the sound wave to the shear waves terminates. The consequences of this effect depend on several factors. If the structure damage is irreversible, the subsequent evolution of the three-wave ensemble is of no interest. If the processes are reversible, an alternation of intervals corresponding to the generation of shear waves and to the absence of generation is possible. This situation should affect the initial sound field, which will propagate in the medium with fluctuating parameters and will experience strong fluctuations itself.

We made an attempt to indirectly observe the aforementioned effect in model experiments. Specimens of agarose hydrogel (this material is conventionally used in practical medicine and biology as a phantom of soft biological tissue) were irradiated with pulsed focused ultrasound. The purpose of the experiments was to observe the amplitude fluctuations of ultrasound that occur above a certain intensity threshold and can be interpreted as parametric generation of shear waves.

The specimens of agarose hydrogel had the form of semitransparent elastic cylinders with a diameter of 30 mm and a length of 29 mm. They were prepared by jellification of an agarose aqueous solution with a mass concentration of 1.5%. In the course of preparation, the specimens were degassed.

The measurements were performed in a partially anechoic tank ($20 \times 56 \times 28 \text{ cm}$) with precipitated water. The experimental setup is schematically represented in Fig. 3. Longitudinal waves were excited in the medium by a focusing piezoceramic transducer. The transducer (J) was fixed to the rod of a traverse gear, which allowed displacements of the transducer in three mutually perpendicular directions, as well as controlled variations of the slope of the transducer axis. The parameters of the focusing transducer were as follows: a frequency of 0.969 MHz, a plate diameter of 62 mm, and a focal length of 70 mm. The calculated dimensions of the focal region in water were: a diameter of 4.4 mm and a length of 16 mm. To minimize the possibility of cavitation on the path of the acoustic

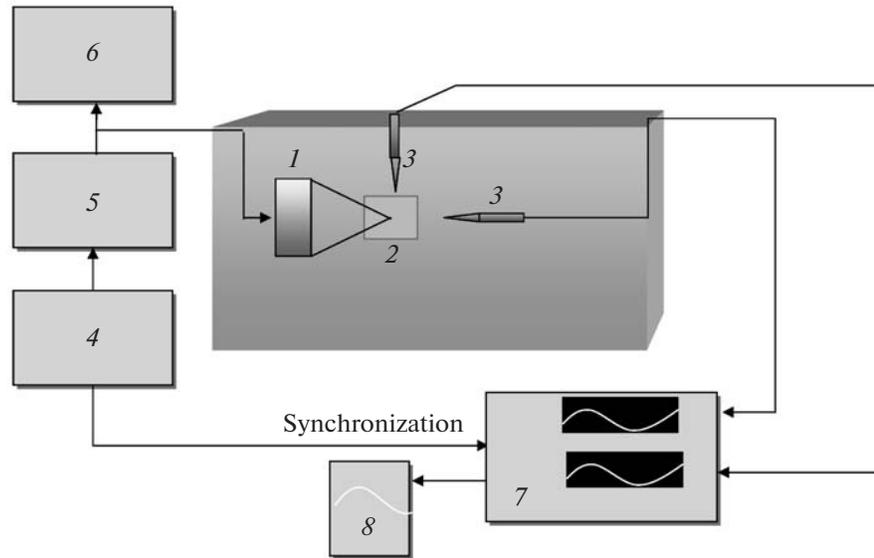


Fig. 3. Experimental setup: (1) focusing transducer (with a frequency of 0.969 MHz, a plate diameter of 62 mm, and a focal length of 70 mm), (2) specimen, (3) hydrophones, (4) precision clock oscillator (G3-110), (5) power amplifier (UZGM 50-dB), (6) monitor millivoltmeter, (7) A/D converter (LA-n4USB), (8) personal computer.

beam in water, the transducer was placed into a truncated cone whose outlet hole was closed with a sound-transparent film, and the cone was filled with degassed water. The specimen under study (2) was mounted on a special base in a “soft” manner at the center of the tank on the axis of the transducer. The end of the specimen was perpendicular to the axis of the sound beam and was pressed to the outlet hole of the cone. The focal region was inside the specimen.

The total acoustic power of the transducer was determined by the measurement of the radiation force in water and varied from 0.66 to 87 W. The calculated value of ultrasound intensity (in water) averaged over the cross section of the focal spot correspondingly varied from 3.5 to 466 W/cm². The specimens were insonified by isolated pulses with a duration of 60 ms at 2-min intervals, while the level of the incident wave was increased from pulse to pulse by 1 dB.

The receivers of acoustic signals were two hydrophones (3), which had the form of small piezoceramic cylinders with a diameter of 1.5 mm; the cylinders were inserted into metal tubes with an outer diameter of 2 mm. The amplitude–frequency characteristic of the hydrophones in the frequency band under study was uniform. The receivers were positioned as follows: one of them was in the field of ultrasound transmitted through the specimen, at the axis of the radiating transducer, at a distance of 20–30 mm from the rear end of the specimen, i.e., at a distance of 45 mm from the center of the focal region; the other receiver was in the focal plane in water, at a distance of 1–3 mm from the surface of the specimen (16–18 mm from the beam axis). The latter hydrophone received the signal in the scattered field. The signal from the hydrophones

was supplied to the input of an 8-digit LA-n4USB digital converter (7), which was connected to a personal computer (8) through a USB bus. The transmission band of the converter was up to 100 MHz, and the input sensitivity was ± 0.125 V. The sampling frequency was 15.625 MHz. The received signals, namely, the transmitted and scattered ones, were visualized on the PC display and saved as two-channel sound files. At the stage of data processing, the average amplitudes of the transmitted and scattered signals were measured and then subjected to spectral analysis in the frequency band from 0.45 to 1 MHz (using the SpectraLAB program package). The received signals were tested for the presence of low-frequency modulation and for the presence of the half-frequency subharmonic component.

Figure 4 shows the squared amplitudes of signals received by the hydrophones in the acoustic field transmitted through the specimen and in the scattered field versus the total acoustic power of the radiating transducer. One can see that the intensity of the transmitted wave increases in direct proportion with the power of incident ultrasound. When the ultrasonic power exceeds 20 W, the slope of the curve representing the growth of the transmitted signal slightly decreases, which testifies to an increase in energy loss along the path of the sound beam. At the same time, the scattered signal, being almost absent in the small power region, begins to exceed the noise level when the power reaches 20 W. As the power increases further, the amplitude of the scattered signal rapidly grows, this growth being much faster than that of the transmitted signal amplitude.

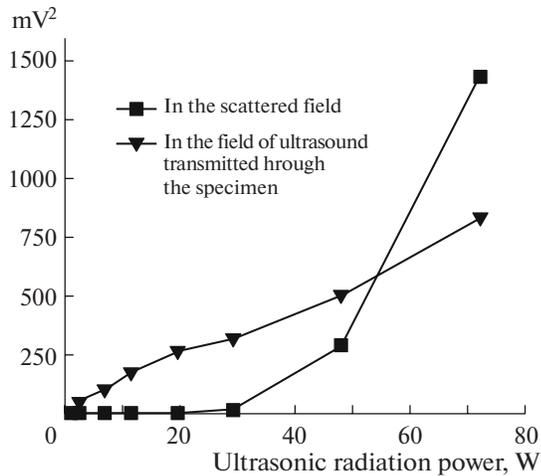


Fig. 4. Squared amplitude of hydrophone signals in the scattered field and in the field of ultrasound transmitted through the specimen versus the radiation power of the transducer.

Figure 5 shows typical records of signals obtained from the hydrophones as a result of insonifying a specimen of agarose gel by a single pulse (60 ms) of focused ultrasound. The left-hand plots represent the signals received in the acoustic field after transmission of ultrasound through the specimen, and the right-hand plots show the signals obtained in the scattered field in the focal plane. From top to bottom (both at the left and at the right), the plots correspond to the following values of the total power of incident ultrasound: (a, b) 2.56, (c, d) 19.5, (e, f) 29, (g, h) 48, and (i, j) 72 W. The abscissa axis represents time (in milliseconds), and the ordinate axis represents the signal amplitudes received by the hydrophones (in millivolts).

The records shown in Fig. 5 demonstrate the formation and the development of the low-frequency modulation of signals in the transmitted and scattered fields. At an incident ultrasonic power of 2.56 W (Figs. 5a, 5b), no modulation of signals is observed. At 19.5 W, a low-frequency modulation appears in the scattered signal (Fig. 5d) within 20 ms after the beginning of insonification, whereas the transmitted signal (Fig. 5c) remains unperturbed. At an ultrasonic power of 29 W, the transmitted signal (Fig. 5e) exhibits a jump-like decrease in the signal amplitude within 5 ms after the beginning of insonification, and then minor traces of amplitude modulation appear; at the same time, the scattered signal (Fig. 5f) exhibits a strong amplitude modulation. At higher ultrasonic powers of 48 and 72 W, a fully developed amplitude modulation can be observed in both the transmitted (Figs. 5g and 5i, respectively) and scattered (Figs. 5h and 5j) signals. The drastic increase in the scattered signal level in Figs. 5h and 5j testifies to the presence of scattering

centers inside the specimen on the path of propagation of the ultrasonic beam.

The experimental results presented above show that the action of focused ultrasound on a gel-type medium with an ultrasonic power exceeding a certain threshold leads to sharp changes in the medium, which manifest themselves in fluctuations of the direct ultrasonic signal and in the appearance of the scattered signal. One of the possible mechanisms may be the parametric instability of the sound field, which leads to shear wave generation. The shear waves destroy the gel structure of the medium, and this leads to an increase in absorption and scattering of sound. The threshold value of the radiation power at which fluctuations of the sound field arise can be estimated as 20 W. Recalculating this value to the intensity at the focal spot, we obtain approximately 112 W/cm^2 . This value is close to the theoretical estimate given above.

In closing, we discuss the basic results of this study and formulate the questions that should be answered before the effect under consideration can be accepted (or rejected) as an important effect in biomedical acoustics.

We theoretically showed that, in soft solids, a sufficiently intense longitudinal (sound) wave may cause generation of shear waves. The frequency of shear waves is half the frequency of the longitudinal wave and the wavelength of shear waves is two to three orders of magnitude smaller than the longitudinal wavelength. The generation threshold depends on two parameters: the nonlinearity coefficient, which describes the dependence of the shear modulus of the medium on pressure, and the attenuation coefficient of shear waves. At present, for both biological tissues and simpler model gel-type media, the values of these parameters in the megahertz frequency range are determined neither experimentally, nor theoretically. Therefore, a reliable determination of the shear wave's generation threshold requires direct measurements of nonlinearity coefficients and loss coefficients in soft solids.

In our model experiments, we observed fluctuations of pulsed focused ultrasound in agar-gelatin gel. The fluctuations can be interpreted as a consequence of partial damage of the gel structure by shear waves. However, we cannot rule out the influence of cavitation and admit that, by now, a unique choice of one of these mechanisms is impossible. A more definite conclusion can be made on the basis of repeating the model experiments under a higher static pressure.

Finally, we note that, generally speaking, actual biological tissue consisting of cells is not homogeneous for shear waves. The size of cells and the shear wavelengths in the megahertz frequency range are on the same order of magnitude. However, it is clear that the

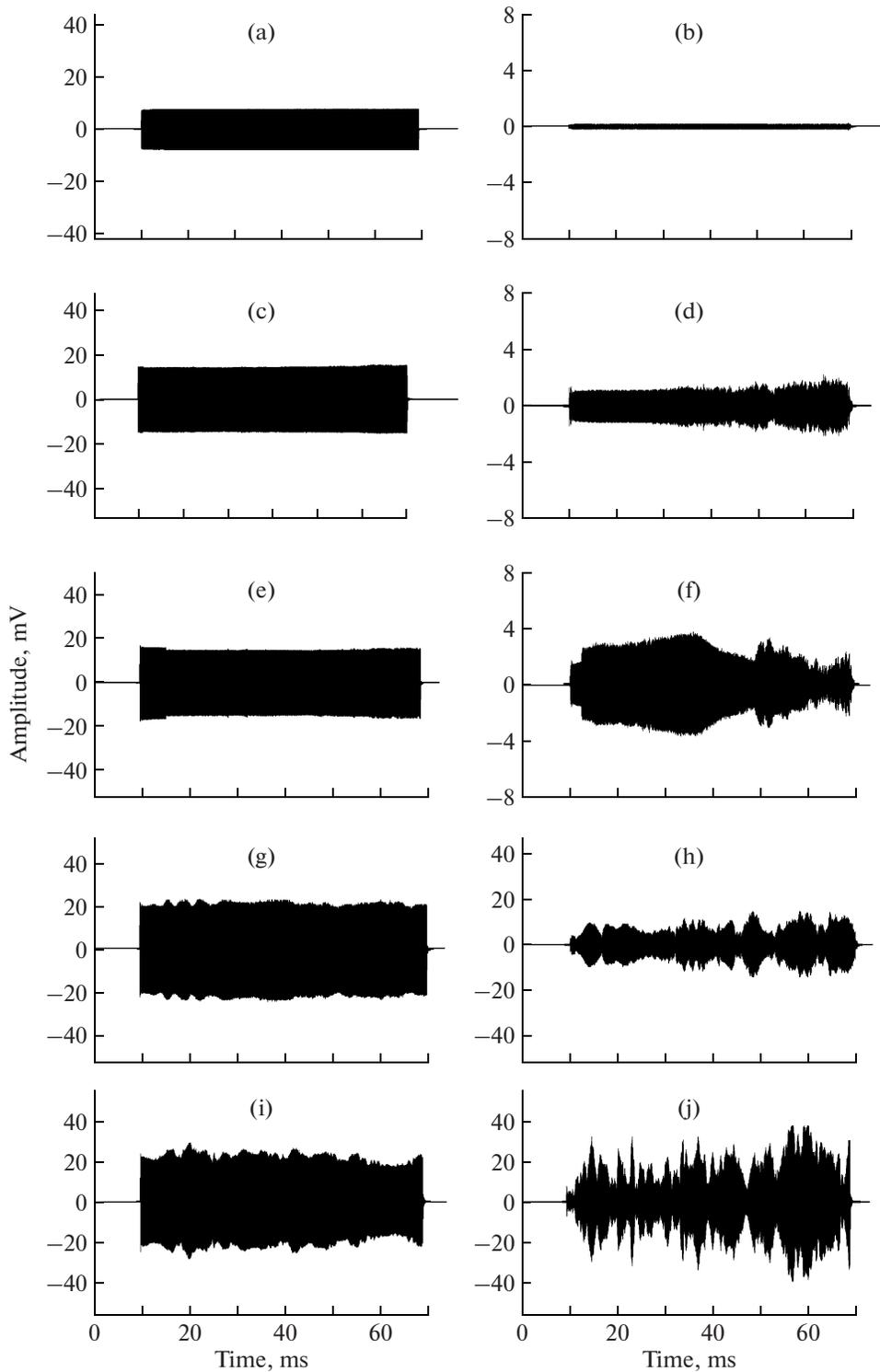


Fig. 5. Records of hydrophone signals obtained by insonifying an agarose gel specimen by a single pulse (60 ms) of focused ultrasound.

inhomogeneity of the medium cannot prevent generation of some other vibrations that may be more complicated than shear waves.

ACKNOWLEDGMENTS

This work was partially supported by the Russian Foundation for Basic Research (project no. 08-03-

00794-a) and the Presidential Program in Support of the Leading Scientific Schools of Russia (grant no. NSh-745.2008.2).

REFERENCES

1. V. G. Andreev, V. N. Dmitriev, Yu. A. Pishchal'nikov, et al., *Akust. Zh.* **43**, 149 (1997) [*Acoust. Phys.* **43**, 123 (1997)].
2. A. P. Sarvazyan, O. V. Rudenko, S. D. Swanson, J. B. Fowlkes, and S. Y. Elemianov, *Ultrasound Med. Biol.* **24**, 1419 (1998).
3. Z. Wua, K. Hoyt, D. J. Rubens, and K. J. Parker, *J. Acoust. Soc. Am.* **120**, 535 (2006).
4. A. V. Vedernikov and V. G. Andreev, *Vestn. Mosk. Univ., Ser. Fiz. Astron.*, No. 3, 52 (2006).
5. L. D. Landau and E. M. Lifshitz, *Course of Theoretical Physics, Vol. 7: Theory of Elasticity* (Nauka, Moscow, 1982; Pergamon Press, New York, 1986).
6. Z. A. Gol'dberg, *Akust. Zh.* **6**, 307 (1960) [*Sov. Phys. Acoust.* **6**, 306 (1960)].
7. G. L. Jones and D. R. Cobett, *J. Acoust. Soc. Am.* **35** (1), 5 (1963).
8. E. A. Zabolotskaya, *Akust. Zh.* **32**, 474 (1986) [*Sov. Phys. Acoust.* **32**, 296 (1986)].
9. O. V. Rudenko and O. A. Sapozhnikov, *JETP* **79**, 220 (1994).
10. M. F. Hamilton, Yu. A. Ilinskii, and E. A. Zabolotskaya, *J. Acoust. Soc. Am.* **116**, 41 (2004).
11. M. S. Wochner, M. F. Hamilton, Yu. A. Ilinskii, and E. A. Zabolotskaya, in *Proc. of the 19th Intern. Congress on Acoustics, Madrid, Spain, 2–7 Sept., 2007*, NLA-03-003.
12. P. A. Pyatakov and M. A. Mironov, in *Proc. of the 16th ISNA* (Moscow, 2002), Vol. 2, p. 815.
13. X. Jacob, S. Catheline, J.-L. Gennisson, et al., *J. Acoust. Soc. Am.* **122**, 1917 (2007).
14. S. Catheline, J.-L. Gennisson, and M. Fink, *Phys. Rev. Lett.* **91**, 43011 (2003).
15. J.-L. Gennisson, M. Rénier, S. Catheline, et al., *J. Acoust. Soc. Am.* **122**, 3211 (2007).
16. W. J. Fry, V. J. Wulf, D. Tucker, and F. J. Fry, *J. Acoust. Soc. Am.* **22**, 867 (1950).
17. W. J. Fry, D. Tucker, F. J. Fry, and V. J. Wulf, *J. Acoust. Soc. Am.* **23**, 364 (1951).
18. F. Dunn, *Am. J. Phys. Med.* **37**, 148 (1958).
19. V. A. Burov, N. P. Dmitrieva, and O. V. Rudenko, in *Proc. of the 16th ISNA* (Moscow, 2002), Vol. 2, p. 411.
20. *Physical Principles of Medical Ultrasonics*, Ed. by C. R. Hill, J. C. Bamber, and G. R. ter Haar (Wiley, London, 2004; Fizmatlit, Moscow, 2008).
21. N. Vykhodtseva, N. McDannold, and K. Hynynen, in *Proc. of the 16 ISNA* (Moscow, 2002), Vol. 2, p. 473.
22. N. Vykhodtseva, N. Dannold, and K. Hynynen, in *Proc. of the 19 Session of RAO* Vol. 3, p. 104 (2007).
23. S. A. Akhmanov and R. V. Khokhlov, *Nonlinear Optics* (VINITI, Moscow, 1964; Gordon and Breach, New York, 1972).

Translated by E. Golyamina